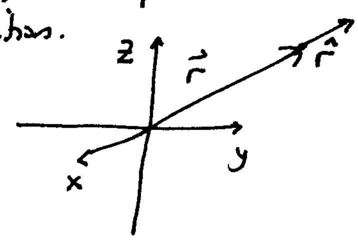
Chem 121
Homework #2

1. Prove that $\vec{\nabla}\cdot(\vec{a}\times\vec{b}) = b\cdot\vec{\nabla}\times\vec{a} - \vec{a}\cdot\vec{\nabla}\times\vec{b}$

2. The electrostatic field of a point charge q is

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\,\frac{\hat{r}}{r^2}$$

, where $\hat{r}$ is a unit vector in the radial direction.



Calculate the divergence of $\vec{E}$. What happens at the origin?

3. Verify that

$$\vec{\nabla}(\vec{A}\cdot\vec{B}) = (\vec{A}\times\vec{\nabla})\times\vec{B} + (\vec{B}\times\vec{\nabla})\times\vec{A} + \vec{A}(\vec{\nabla}\cdot\vec{B}) + \vec{B}(\vec{\nabla}\cdot\vec{A})$$

4. If $\vec{V} = \hat{x}\,V_x(x,y) + \hat{y}\,V_y(x,y)$ and $\vec{\nabla}\times\vec{V} \neq 0$

Show that:

$$\vec{\nabla}\times\vec{V} \perp \vec{V}.$$

5. Verify that:

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{V}) = \vec{\nabla}\,\vec{\nabla}\cdot\vec{V} - \vec{\nabla}\cdot\vec{\nabla}V$$

Note that the position of the dot product makes all the difference!

6. Calculate the work done against the force field

$$\vec{F} = \frac{-\hat{x}y + \hat{y}x}{x^2 + y^2}$$

by calculating the line integrals $-\int \vec{F}\cdot d\vec{r}$ along two paths on the unit circle

(a) going counter clockwise from 0 to π

(b) going clockwise from 0 to $-\pi$

7. Given that $\vec{\nabla}\times\vec{F} = \vec{\nabla}\times\vec{G}$ for two vector fields $\vec{F}$ and $\vec{G}$, are these fields equal? To answer this question, consider $\vec{F}-\vec{G}$. Is this vector field constant, or not. If not, what properties does it have.

8. If $\vec{B} = \vec{\nabla}\times\vec{A}$ show that $\oint_S \vec{B}\cdot d\vec{a} = 0$

for any closed surface S.

9. Recall that the electric potential and static electric field are related by $\vec{E} = -\vec{\nabla}\varphi$. Electrostatic fields are generated by charge densities ρ according to the equation

$$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0 \quad \text{, where } \varepsilon_0 \text{ is a constant.}$$

Prove that

$$\int \rho\varphi \, d^3x = \varepsilon_0 \int E^2 \, d^3x$$

, where the volume integrals extend over all space. You may assume that $\varphi \to 0$ fast enough at $r \to \infty$.