Chem 121
Homework #1

1. The vector $\vec{r}$ starting at the origin terminates at and specifies the point in space (x, y, z). Find the surface swept out by $\vec{r}$ if

(a) $(\vec{r}-\vec{a})\cdot\vec{a} = 0$

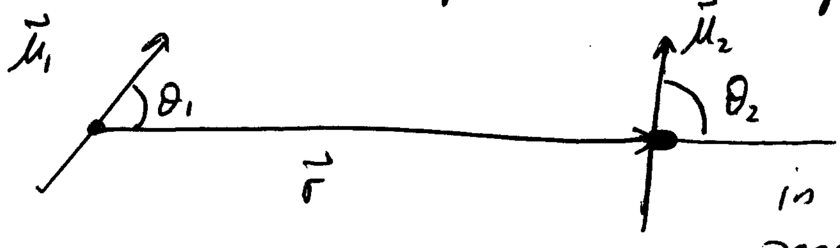
(b) $(\vec{r}-\vec{a})\cdot\vec{r} = 0$

(c) $\vec{r}\cdot\vec{r} - a^2 = 0$

$\vec{a}$ is a constant vector

2. The interaction energy between two dipoles $\vec{\mu}_1$ and $\vec{\mu}_2$ may be written as

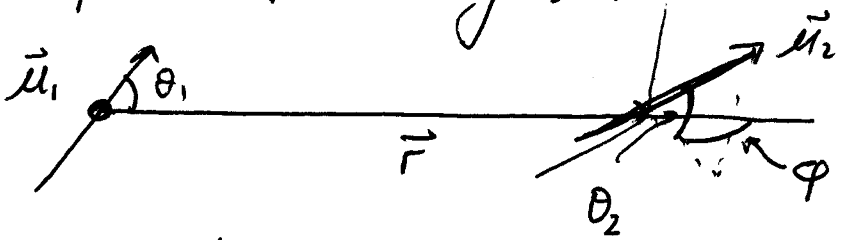
$$V = -\frac{\vec{\mu}_1\cdot\vec{\mu}_2}{r^3} + \frac{3(\vec{\mu}_1\cdot\vec{r})(\vec{\mu}_2\cdot\vec{r})}{r^5}$$



(both dipoles are in the plane of the paper)

(a) Find the interaction energy in terms of the angles θ_1 and θ_2

(b) Now suppose that $\vec{\mu}_2$ does <u>not</u> lie in the plane spanned by $\vec{\mu}_1$ and $\vec{r}$.



Find the interaction energy in terms of θ_1, θ_2 and φ.

3. For two vectors $\vec{A}$ and $\vec{B}$ prove that

$$(\vec{A}\times\vec{B})\cdot(\vec{A}\times\vec{B}) = A^2B^2 - (\vec{A}\cdot\vec{B})^2$$

For three vectors $\vec{A}, \vec{B}, \vec{C}$ show that

$$\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$$

4. Find the angles and sides of a spherical triangle ABC defined by the vectors

$$\vec{A} = (1,0,0); \quad \vec{B} = \frac{1}{\sqrt{2}}(1,0,1); \quad \vec{C} = \frac{1}{\sqrt{2}}(0,1,1)$$

Draw a picture of your triangle.

5. The magnetic induction field $\vec{B}$ is defined by its action on a moving charge. The force $\vec{F}$ is given by

$$\vec{F} = q\,(\vec{v}\times\vec{B})$$

for a charge q moving with velocity $\vec{v}$.

You have performed three experiments to measure the force on this charge

(i) $\vec{V} = \hat{x} \Rightarrow \dfrac{\vec{F}}{q} = 2\hat{z} - 4\hat{y}$

(ii) $\vec{V} = \hat{y} \Rightarrow \dfrac{\vec{F}}{q} = 4\hat{x} - \hat{z}$

(iii) $\vec{V} = \hat{z} \Rightarrow \dfrac{\vec{F}}{q} = \hat{y} - 2\hat{x}$

Find the magnetic field $\vec{B}$.

6. The vector $\vec{D}$ is a linear combination of the vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$:

$$\vec{D} = a\vec{A} + b\vec{B} + c\vec{C} \quad ; \ a, b, c \text{ are scalars.}$$

Show that $a = \dfrac{\vec{D}\cdot(\vec{B}\times\vec{C})}{\vec{A}\cdot(\vec{B}\times\vec{C})}$

Find b, c similarly.

7. If $S(x,y,z) = (x^2+y^2+z^2)^{-3/2}$ Find

(a) the vector field $\vec{\nabla} S$

(b) The magnitude $|\vec{\nabla} S|$

④

8.(a) If a function $\vec{F}(x,y,z,t)$ depends on both space (x,y,z) and time t show that

$$d\vec{F} = (d\vec{r}\cdot\vec{\nabla})\vec{F} + \frac{\partial \vec{F}}{\partial t}\,dt$$

(b) You are driving in the mountains along a path

$$\vec{r} = (v_0 t,\ w_0 t,\ z_0 t + a t^2)$$

The temperature in the mountains is a function of both space and time

$$T(x,y,z,t) = \left[T_0 - \alpha\left(\tfrac{1}{2}x - y\right)\right]e^{-z/l} - \beta t$$

α, β, and l are constants.

You have a thermometer in your car. Predict the change in temperature that you read τ as a function of time. It is sufficient to determine

$$\frac{d\tau}{dt}$$

If you want to try to integrate this to get $\tau(t)$ you can. Is there an easier way?