Chem 121
Homework #5

1. Show that $\frac{1}{2\pi i}\oint z^{m-n-1}\,dz \quad m,n \in \mathbb{Z}$ (integers)

with a contour encircling the origin once counterclockwise is a representation of the Kronecker delta $\delta_{m,n}$

2. Using the Cauchy integral equation for the n^{th} derivative of a function, convert the Rodrigues formula for the Legendre polynomials into the Schlaefli integral form:

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n \quad \text{Rodrigues formula}$$

$$\frac{(-1)^n}{2^n}\frac{1}{2\pi i}\oint \frac{(1-z^2)^n}{(z-x)^{n+1}}\,dz$$

Understanding how to do this helps you develop the standard generating functions for all the special functions. You can try the same trick as above with the Hermite functions for the harmonic oscillator as another example.

3. Find the singularities of the following functions. What kind are they?

a) $\dfrac{1}{z^2 + a^2}$

c) $\dfrac{\sin(1/z)}{z^2 + a^2}$

b) $\dfrac{1}{(z^2+a^2)^2}$

d) $\dfrac{z^{-k}}{z+1}$ $\quad 0 < k < 1$

← be careful here and think about an example w/ logs!

4. Show that $\displaystyle\int_0^\infty \frac{dx}{(x^2+a^2)^2} = \frac{\pi}{4a^3}$, $a > 0$

5. Calculate $\displaystyle\int_{-\infty}^{\infty} \frac{x^2}{1+x^4}\,dx$ by contour integration

6. A quantum mechanical calculation of a transition probability leads to $f(\omega,t) = \dfrac{2(1-\cos(\omega t))}{\omega^2}$

Show that $\displaystyle\int_{-\infty}^{\infty} f(\omega,t)\,d\omega = 2\pi t$