Chem 121
Homework #3

1. If A is an $n \times n$ matrix show that

$$\det(-A) = (-1)^n \det(A)$$

2. Given the matrices $A = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ $B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $C = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

Find all possible products of A, B, C (two at a time) and express your answer in terms of A, B, C, and $\mathbb{1}$ ← the identity matrix.

You will have constructed the multiplication table for the vierergruppe (D_2 in more common notation).

3. Given $K = \begin{pmatrix} 0 & 0 & i \\ -i & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}$ show that

$K^n = \underline{\mathbb{1}}$ for some choice of $n \neq 0$.

4. A and B are anti-commuting matrices such that $A^2 = \mathbb{1}$ and $B^2 = \mathbb{1}$.

Show that $\mathrm{Tr}\,A = \mathrm{Tr}\,B = 0$

Note that Pauli and Dirac matrices are common examples of this.

5. Show that the sum of squares of the elements of a matrix remains invariant under orthogonal similarity transformations.

6. If A is a 2×2 matrix show that its eigenvalues λ satisfy the equation

$$\lambda^2 - \lambda \,\mathrm{Tr}(A) + \det(A) = 0$$

7. Find the eigenvalues of a projection operator P. A projection operator satisfies the condition
$$P^2 = P$$
Do you see why?

8. A is a Hermitian $n\times n$ matrix with eigenvectors $|r_i\rangle$, i.e.

$$A|r_i\rangle = \lambda_i |r_i\rangle \qquad i=1,\ldots,n$$

Say $|r\rangle$ is an approximation to $|r_1\rangle$. It can be expanded in the basis of A's eigenvectors:

$$|r\rangle = |r_1\rangle + \sum_{i=2}^{n} \delta_i |r_i\rangle \qquad \underline{(\text{presumably } |\delta_i| \ll 1)}$$

show that

$$\frac{\langle r|A|r\rangle}{\langle r|r\rangle} \leq \lambda_1 \quad \text{and that}$$

∴ the error in λ_1 (computed above) is
order $|\delta_i|^2$

Thus a bad guess at a quantum state can yield a
better guess at (say) its energy eigenvalue!