Suppression of the Josephson effect by quantum fluctuations in the fractional quantum Hall state

Shechao Feng, Alex Levine, and Lan Yin
Department of Physics, University of California, Los Angeles, California 90024-1547

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We use the Chern-Simons gauge theory for the fractional quantum Hall effect (FQHE) to calculate the Josephson current for two weakly coupled two-dimensional electron gas (2DEG) subsystems separated by a thin tunnel barrier, for the primary fractional quantum Hall states $\nu = 1/(2k+1)$ ($k = 0, 1, 2, ...$), at zero temperature. We find that the Josephson current amplitude is mainly suppressed by the long-range quantum fluctuations of the Chern-Simons gauge field, by a factor $\propto \exp(-\frac{2k+1}{4\pi}\frac{L}{l_0})$, where L is the system size, and l_0 is of the order of the magnetic length. Thus it is unlikely that the Josephson effect can be observed for weakly coupled 2DEG subsystems, despite the strong analogy between the FQHE in the Chern-Simons formulation and two-dimensional superconductivity.

I. INTRODUCTION

Recently a theoretical framework has been developed to describe the fractional quantum Hall effect (FQHE) in two-dimensional electron gas (2DEG) systems, based on an exact Chern-Simons gauge transformation, which maps the original interacting electron system in a high magnetic field to that of interacting Chern-Simons (CS) bosons under an (on average) zero magnetic field.[1,2] To put it simply, this approach treats an electron plus $2k+1$ flux quanta as a composite (Chern-Simon) "particle" which can be shown to obey Bose statistics. For the primary FQHE states $\nu = 1/(2k+1)$, it is easy to show that the CS bosons on average do not see a net magnetic field. At the mean-field level, one then argues that these CS bosons condense into a superfluid state at low enough temperatures, and the (dissipationless) quantum Hall effect is equivalent to the superfluidity (or superconductivity) of the charged CS bosons, which play a similar role as the Cooper pairs in the problem of superconductivity. It can be shown that if one starts from the mean-field theory of these superfluid CS bosons, and then include the effect of quantum fluctuations calculated to (quadratic) Gaussion order, one can reproduce exactly the original Laughlin wave function for the primary fractional quantum Hall state $\nu = 1/(2k+1)$.[2] Thus this CS boson theory of FQHE is equivalent to the Laughlin variational approach.[3] What this approach offers in addition to the standard Laughlin variational approach is that to a large extent, the phenomenon of fractional quantum Hall effect can be mapped onto the phenomena of superfluidity and superconductivity (for the Chern-Simons bosons), which are familiar and well studied subjects. Thus, the relationship between the CS gauge theory and the Laughlin wave function theory for FQHE is quite analogous to that between the Ginzburg-Landau formalism and the BCS variational wave function approach in the theory of superconductivity. The Gaussian fluctuations of the CS gauge field can be also shown to lead to the algebraic decay of the bosonic order parameter, which was discovered soon after Laughlin's variational wave function was first developed.[4]

Several interesting theoretical concepts have been developed on the FQHE using this new Chern-Simons approach to FQHE. One notable example is the prediction of the Hall insulator state at the primary filling factor $\nu = 1/(2k+1)$, namely, as one increases disorder in the system, one finds $\rho_{xx} \to \infty$, whereas $\rho_{xy} \to$ constant.[5] The various mapping rules between the various different fractional Hall states as well as the integer quantum Hall states, and the global phase diagram for the FQHE has also been developed using the Chern-Simons approach,[5] following the original formulation by Jain.[6]

We also note that the Chern-Simons gauge theory has been recently applied to the even denomenator state (e.g., $\nu = 1/2$) with great success. In this case, the composite CS particle consists of an electron plus two flux quanta, which still obeys Fermi statistics. At $\nu = 1/2$, the CS fermions on average do not see a net magnetic field. Thus the transport properties of the $\nu = 1/2$ quantum Hall state can be mapped to a large extent to that of a simple fermi liquid at zero magnetic field.[7] Such an illuminating picture for the transport properties at $\nu = 1/2$ has recently received much support from several experiments.[8]

In this paper, we take the logical next step in exploring the similarities between the FQHE and superconductivity, namely, to calculate the Josephson effect in a weak-link FQHE system using the Chern-Simons approach. We envision our weak-link 2DEG system to consist of two sheets of electron gas (with equal dimensions $L_x \times L_y$), which are coupled weakly at $x = 0$ (see Fig. 1). We assume for simplicity that the area of the junction itself has zero area, so that the "diffraction" type reduction of the Josephson current by the magnetic flux through the junction area can be neglected. In theory, such a weak-link geometry can be achieved by growing (using MBE) a

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$GaAs/Al_xGa_{1-x}As$ heterostructure on top of a disorder-free substrate which contains an atomically sharp step, as we depict in Fig. 1. Since the single electron wave function is confined to a short distance λ ($\lambda \sim 70$ Å) in the vertical z direction, an appropriate choice of the step size $d > \lambda$ would result in a weak coupling between the two 2DEG subsystems (hereafter we label them 1 and 2). The task of the following sections is to use the Chern-Simons gauge theory to calculate the Josephson current for the system just proposed.

Our main conclusions are the following: to mean-field order, we found a sizable Josephson current amplitude between two weakly coupled 2DEG subsystems. But going beyond the mean-field order, we found that the dominant quantum fluctuations which suppress the Josephson effect come from that of the phase factor relating the Chern-Simons field operator and that of an electron, $\theta^{\rm CS}$. Fluctuations of $\theta^{\rm CS}$ in the (Laughlin) ground-state wave function produces a suppression factor to the mean-field Josephson current amplitude of the form $\propto \exp(-\frac{2k+1}{4\pi}\frac{L}{l_0})$, where L is the system size, and l_0 is of order the magnetic length. The presence of such a severe quantum suppression factor makes any experimental observation of the Josephson effect between two weakly couple FQH subsystems extremely difficult, if not impossible. We regard this severe suppression of the Josephson effect as a crucial difference between the physics of fractional quantum Hall states and two-dimensional superconductors, despite many similarities between the two problems, when the FQHE is formulated in terms of Chern-Simons bosons.

We shall organize the paper in the following way. In Sec. II, we provide the basic formulation of the problem of Josephson current through a weak-link junction, using the Chern-Simons approach. We will then proceed to give the results in mean-field theory. In Sec. III, we calculate the quantum fluctuations which give rise to the exponential suppression factor to the mean-field result. In Sec. IV, we discuss implications of our results to other geometries of weakly coupled FQHE subsystems, and make some concluding remarks.

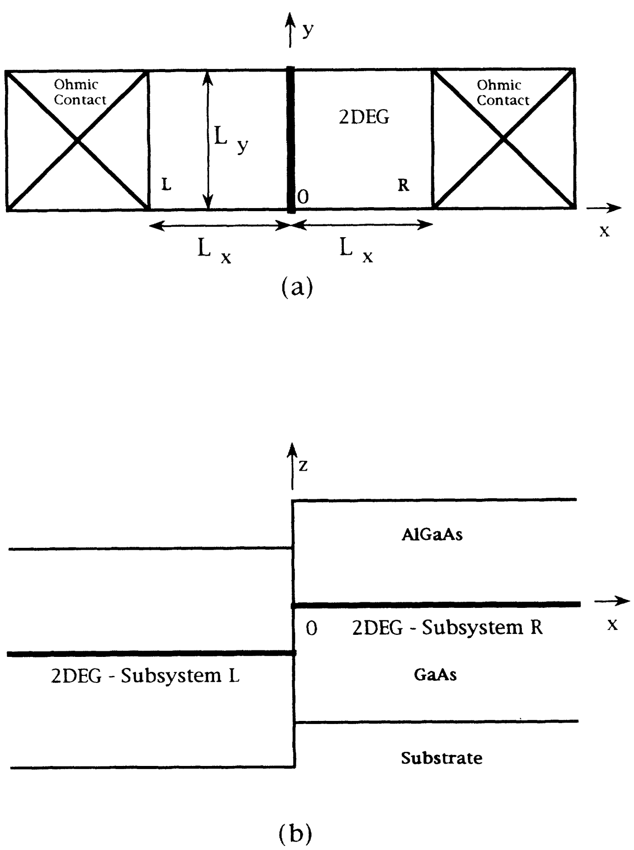

FIG. 1. The geometry of the weak-link 2DEG system under study. We assume that a high mobility $GaAs/Al_xGa_{1-x}As$ heterostructure system is grown on top of a disorder-free substrate which contains an atomically sharp step of height d at $x = 0$. (a) Top view of the device; (b) side view of the device. The two 2DEG subsystems are assumed to have the same dimensions L_x and L_y.

II. FORMULATION OF THE PROBLEM AND THE MEAN-FIELD SOLUTION

We confine our calculation to the primary fractional Hall states which have filling factors $\nu = 1/(2k+1)$, with $k = 0, 1, 2, \ldots$. Thus in our formulation, the integer quantum Hall effect at $\nu = 1$ is treated on an equal footing as $\nu = 1/3$, for example. The Hamiltonian for this weak-link FQHE system in terms of the original electron (fermion) field operators $\psi_L(\vec{r})$ and $\psi_R(\vec{r})$ in subsystems L (for left) and R (for right) consists of three contributions:

$$H = H_L + H_R + H_T. \qquad (1)$$

The Hamiltonian for subsystem L ($x < 0$) takes the usual form

$$H_L = \frac{1}{2m^*}\int_{x<0} d^2x \psi_L^\dagger(\vec{x}) \left(\frac{\hbar}{i}\vec{\nabla} + \frac{e}{c}\vec{A}(\vec{x})\right)^2 \psi_L(\vec{x}) + \frac{1}{2}\int_{x<0} d^2x d^2y \delta\rho_L(\vec{x}) V(\vec{x}-\vec{y}) \delta\rho_L(\vec{y}), \qquad (2)$$

where $\nabla \times \vec{A}(\vec{x}) = B\hat{z}$ is the uniform external magnetic field, $\vec{x} = (x, y)$ are the spatial coordinates, $V(\vec{x})$ is the (Coulomb) interaction potential among electrons, and $\delta\rho_L(\vec{x}) \equiv \rho_L(\vec{x}) - \bar{\rho}$, where $\rho_L(\vec{r}) = \psi_L^\dagger(\vec{r})\psi_L(\vec{x})$ is the electron density operator, and $\bar{\rho} = \nu B/\phi_0$ is the average electron density which we assume to be the same in both subsystems L and R.

H_R can be obtained from Eq. (2) by substituting $L \to R$, and the integrations are over the region $x > 0$ in this case.

The tunneling Hamiltonian H_T in Eq. (1) is the most important part of the total Hamiltonian which provides the weak coupling between the two subsystems at $x = 0$. For the weak-link geometry we consider here, we write H_T in terms of the original electron operators in the following way:

$$H_T = E_0 \sum_{\vec{k}} (T a_{\vec{k}}^\dagger b_{\vec{k}} + \text{H.c.}) \;, \qquad (3)$$

where $a_{\vec{k}}^\dagger$ is the electron creation operator in the left (L) region ($x < 0$) for momentum mode $\vec{k}$, and $b_{\vec{k}}^\dagger$ is that in the right (R) region. We have here assumed implicitly that the tunnel junction between the two 2DEG subsystems is perfectly flat (in the y direction), so that the transverse momentum of the tunneling electron is conserved upon tunneling from one side of the barrier to the other. Since the tunneling process also conserves energy, the longitudinal component of electron's momentum must also be conserved. Therefore, only the same (vector) momentum states are coupled through our tunneling Hamiltonian Eq. (3).

We take here E_0 as some characteristic energy scale for the step perturbation potential at the junction, which should be of the order of the cyclotron energy $\hbar\omega_c$, and $T \sim e^{-d/\lambda} \ll 1$ is a dimensionless overlap integral (amplitude transmittance) of the z-directional wave function from the two subsystems separated by a sharp step (see Fig. 1), or is due to some other type of tunnel barrier between the two 2DEG subsystems.

Since we have the relation

$$\psi_L(\vec{x}) = \frac{1}{\sqrt{A}} \sum_{\vec{k}} e^{i\vec{k}\cdot\vec{x}} a_{\vec{k}},$$

for $x < 0$ with $A = L_x L_y$, and

$$\psi_R(\vec{x}) = \frac{1}{\sqrt{A}} \sum_{\vec{k}} e^{i\vec{k}\cdot\vec{x}} b_{\vec{k}},$$

for $x > 0$, we can rewrite the tunneling Hamiltonian H_T in the following simple form:

$$H_T = E_0 T \int_{x<0} d^2x \psi_L^\dagger(\vec{x}) \psi_R(\vec{x} + L\hat{x}) + \text{H.c.} \qquad (4)$$

We see clearly that the tunneling Hamiltonian for a flat (i.e., disorder free) tunnel barrier gives rise to a coupling between any given point $\vec{x}$ in the *bulk* of the left subsystem and the corresponding point $\vec{x} + L\hat{x}$ in the right subsystem, and vice versa. Thus, it is our opinion that for the problem of coherent tunneling between the two 2DEG subsystems, the relevant physics is that of the bulk rather than the edge. The edge states are important for the dissipative (or incoherent) tunneling channel between the two subsystems, as discussed recently by Kane and Fisher.[9]

We can now perform the (exact) Chern-Simons gauge transformation in subsystems L and R separately. In subsystem L, we introduce the CS boson field operator as

$$\phi_L(\vec{x}) = e^{i\theta_L^{\rm CS}(\vec{x})} \psi_L(\vec{x}), \qquad (5)$$

with

$$\theta^{\rm CS}(\vec{x})_L \equiv m \int_{x<0} d^2y \alpha(\vec{x} - \vec{y}) \rho_L(\vec{y})$$

being the Chern-Simons phase angle. Here, $m = 2k + 1$ is the odd integer corresponding to the primary filling factor $\nu = 1/(2k + 1)$, $\alpha(\vec{x} - \vec{y})$ is the azimuthal angle substended by the vector $\vec{x} - \vec{y}$ with respect to the x axis, and $\phi_0 = hc/e$.

It is easy to verify that the field operator $\phi_L(\vec{x})$ satisfies Bose statistics:

$$[\phi_L(\vec{x}), \phi_L^\dagger(\vec{y})] = \delta(\vec{x} - \vec{y}).$$

The physical meaning of the Chern-Simons transformation is to tie $2k + 1$ flux quanta to each electron dynamically, such that a given electron at position $\vec{x}$ "feels" the Aharonov-Bohm phase factor from the flux quanta tied to all the other electrons. Since we tie an odd number of flux quanta to each electron at filling factor $\nu = 1/(2k + 1)$, an exchange operation between two electrons introduce an additional statistical factor $(-1)^{2k+1}$ in addition to the usual Fermi exchange factor (-1), thus making the ϕ operator a bosonic one.

The Hamiltonian H_L in terms of the CS boson field operators takes the form

$$H_L = \frac{1}{2m^*} \int_{x<0} d^2x \phi_L^\dagger(\vec{x}) \left(\frac{\hbar}{i}\vec{\nabla} + \frac{e}{c}\vec{A}(\vec{x}) - \vec{a}_L(\vec{x}) \right)^2 \phi_L(\vec{x}) + \frac{1}{2} \int_{x<0} d^2x d^2y \delta\rho_L(\vec{x}) V(\vec{x} - \vec{y}) \delta\rho_L(\vec{y}), \qquad (6)$$

where $\vec{a}_L(\vec{x})$ satisfies the CS constraint relation (corresponding to the physical requirement that each electron is tied to $2k + 1$ flux quanta):

$$b_L(\vec{x}) \equiv (\nabla \times \vec{a}_L)_z = (2k + 1)\phi_0 \rho_L(\vec{x}). \qquad (7)$$

Here, we can write $\rho_L(\vec{x}) = \phi_L^\dagger(\vec{x})\phi_L(\vec{x})$ in the new CS representation.

It is clear that the CS bosons in the L region feel an effective "magnetic field" given by

$$\delta b_L \equiv b_L - B = (\nabla \times \delta\vec{a}_L)_z = (2k + 1)\phi_0 \delta\rho_L(\vec{x}), \qquad (8)$$

where $\delta\rho_L(\vec{x}) = \rho_L(\vec{x}) - \bar{\rho}$, at primary filling factor $\nu = 1/(2k + 1)$. Thus, we see explicitly that upon averaging over the sample area, the effective magnetic field felt by the CS bosons $\delta\bar{b}_L = 0$. The physical magnetic field B has been "gauged away" in an average sense by the Chern-Simons canonical transformation. As we shall see, however, that quantum fluctuations in δb_L have important consequences, one of which is the severe suppression of the Josephson effect which would be present if the system were to be describable by the mean-field theory.

We can perform the Chern-Simons transformation for subsystem R identically as above, with the notational change $L \to R$. We emphasize here that we have chosen to perform the CS transformation separately for the L and R regions, with two CS angular variables [Eq. (5)]. The justification is that this is the natural zeroth order solution when the coupling between the two subsystem

$T \to 0$. As we are mainly interested in the small T limit, this is the most convenient basis to do calculations. In principle, we could have chosen to introduce a single CS angular variable, with an integration over both the L and R regions. But this formalism would couple the L and R Hamiltonians even when $T \to 0$, thus the calculations we are about to perform will become much more convoluted. We expect, however, that our final results will not be affected even if this alternative CS transformation was used.

We now consider how the tunneling Hamiltonian is modified under the CS transformation. It now takes the form

$$H_T = E_0 \int_{x<0} d^2x \left\{ T\phi_L^\dagger(\vec{x}) \exp[i\theta_L^{\rm CS}(\vec{x})] \times \phi_2(\vec{x}+L\hat{x}) \exp[-i\theta_R^{\rm CS}(\vec{x}+L\hat{x})] + {\rm H.c.} \right\}. \quad (9)$$

It is important to notice that in addition to the boson field operators, we have the additional Chern-Simons phase factors which we shall show lead to severe suppression of the Josephson current amplitude.

The operator form of the Josephson current at zero temperature can be derived from

$$\begin{aligned} I_J = \dot{Q}_L &= e\frac{d}{dt}\int_{x<0} d^2x \rho_L(\vec{x}) \\ &= \frac{e}{i\hbar}\int d^2x [\phi_L^\dagger(\vec{x})\phi_L(\vec{x}), H] = \frac{e}{i\hbar}\int d^2x[\phi_L^\dagger(\vec{x})\phi_L(\vec{x}), H_T] \\ &= \frac{2eE_0}{\hbar}\int_{x<0} d^2x {\rm Im}\left\{T\phi_L(\vec{x})\exp[i\theta_L^{\rm CS}(\vec{x})] \times \phi_R^\dagger(\vec{x}+L\hat{x})\exp[-i\theta_R^{\rm CS}(\vec{x}+L\hat{x})]\right\}. \end{aligned} \quad (10)$$

Within the mean-field theory, we make the approximation $b_{L(R)}(\vec{x}) \to \bar{b} = B$, thus the CS bosons are not subject to any effective magnetic field in both subsystems ($\delta\bar{b}_{L(R)} = 0$). Thus we may take the average CS bosonic fields (order parameters of the quantum Hall liquid) to be simply C numbers, in the form of $\bar{\phi}_L = \sqrt{\bar{\rho}}e^{i\bar{\theta}_L}$, and $\bar{\phi}_R = \sqrt{\bar{\rho}}e^{i\bar{\theta}_R}$, where $\bar{\theta}_L$ and $\bar{\theta}_R$ are two mean-field phases, the difference of which drives a coherent Josephson current across the tunnel barrier, similar to the case of two weakly coupled superconductors. Since we assume the two subsystems to be identical in shape, we also have the relation $\bar{\theta}_L^{\rm CS}(\vec{x}) = \bar{\theta}_R^{\rm CS}(\vec{x}+L\hat{x})$ at the mean-field level. Thus, the messy CS phase factors in Eq. (10) simply drops out in the mean-field theory of the Josephson effect for FQHE states. This immediately leads to the first Josephson relation in mean-field theory as (assuming for simplicity that the overlap integral T is a real number)

$$I_J^{\rm MF} = I_0^{\rm MF}\sin(\bar{\theta}_L - \bar{\theta}_R), \quad (11)$$

where the Josephson current amplitude in the mean-field theory is given by

$$I_0^{\rm MF} = \frac{2eTE_0\bar{\rho}A}{\hbar}. \quad (12)$$

In analogy with the superconductors, the second Josephson relation is given by

$$\hbar\frac{d}{dt}(\bar{\theta}_L - \bar{\theta}_R) = eV, \quad (13)$$

where V is an applied voltage between the two subsystems L and R.

In the next section, we shall see how quantum fluctutations of the $\theta^{\rm CS}$ angular variables in the ground-state (Laughlin) wave function suppress severely the above mean-field result for the Josephson current, which makes it essentially impossible to observe the coherent channel of quantum tunneling between two weakly coupled quantum Hall liquids.

III. SUPPRESSION OF JOSEPHSON CURRENT AMPLITUDE BY QUANTUM FLUCTUTATIONS

It is crucial to see how quantum fluctuations suppress the mean-field result from the previous section, i.e., we need to calculate

$$I_J = \frac{2eE_0T}{\hbar}\int_{x<0} d^2x {\rm Im}\left\{ \langle\phi_L(\vec{x})\exp[i\theta_L^{\rm CS}(\vec{x})]\rangle_{H_L} \times \langle\phi_R^\dagger(\vec{x}+L\hat{x})\exp[-i\theta_R^{\rm CS}(\vec{x}+L\hat{x})]\rangle_{H_R} \right\}, \quad (14)$$

where $\langle\,\rangle_{H_L}$ denotes the quantum expectation value ($T = 0$) over the ground-state (Laughlin) wave function for subsystem L, which is the ground-state eigenfunction of H_L to Gaussian approximation (similarly for subsystem R). Introducing fluctuational variables $\delta\rho(\vec{x})$ and $\delta\theta(\vec{x})$ (omitting the subscript L or R from now on as it is clear from context),

$$\phi(\vec{x}) = \sqrt{\bar{\rho} + \delta\rho(\vec{x})}e^{i[\theta_1 + \delta\theta(\vec{x})]},$$

expand H_L up to quadratic order in $\delta\rho(\vec{x})$ and $\delta\theta(\vec{x})$, and perform Fourier transforms $\delta\rho(\vec{x}) = \frac{1}{A}\sum_{\vec{q}} e^{i\vec{q}\cdot\vec{x}}\delta\rho_{\vec{q}}$, etc., we can write for the CS statistical gauge field in Fourier space as

$$\delta a_{\vec{q}}^{\alpha} = i(2k+1)\phi_0\epsilon^{\alpha\beta}q_\beta\delta\rho_{\vec{q}}/q^2,$$

using the transverse gauge $\nabla\cdot\delta\vec{a}(\vec{x}) = 0$. We then arrive at the following simple expression for H_L:[2]

$$H_L = \text{const} + \frac{\hbar^2\bar{\rho}}{2m^*A}\sum_{\vec{q}}\left\{\left(\frac{[2\pi(2k+1)]^2}{q^2} + \frac{m^*V_{\vec{q}}}{\hbar^2\bar{\rho}} + \frac{q^2}{\bar{\rho}^2}\right)\delta\rho_{\vec{q}}\delta\rho_{-\vec{q}} + q^2\delta\theta_{\vec{q}}\delta\theta_{-\vec{q}}\right\}, \quad (15)$$

where $\delta\rho_{\vec{q}}$ and $\delta\theta_{\vec{q}}$ obey canonical commutation relation $[\delta\rho_{\vec{q}}, \delta\theta_{\vec{p}}] = iA\delta_{\vec{q},-\vec{p}}$. Thus, for each $\vec{q}$ mode (with $q_x \geq 0$), the quantum fluctuational Hamiltonian takes the simple form of two independent harmonic oscillators, up to quadratic (Gaussian) approximation which we have made here.

After some straightforward but tedious algebra, we can identify the dominant fluctuations in the quantity of interest, $\langle\phi_L(\vec{x})\exp[i\theta_L^{\rm CS}(\vec{x})]\rangle_{H_L}$, to be from the $\delta\theta_L(\vec{x})$ variables and the $\delta\theta_L^{\rm CS}(\vec{x})$ variables. Thus, we may write

$$\langle\phi_L(\vec{x})\exp[i\theta_1^{\rm CS}(\vec{x})]\rangle_{H_L} = \sqrt{\bar{\rho}}e^{i\bar{\theta}_L}\exp\{-\tfrac{1}{2}\langle\delta\theta_L(\vec{x})\delta\theta_L(\vec{x})\rangle - \tfrac{1}{2}\langle\delta\theta_L^{\rm CS}(\vec{x})\delta\theta_L^{\rm CS}(\vec{x})\rangle\}. \quad (16)$$

The fluctuations in the phase of the order parameter, $\frac{1}{2}\langle\delta\theta_L(\vec{x})\delta\theta_L(\vec{x})\rangle$, can be readily evaluated, using the harmonic oscillator Hamiltonian in Eq. (15), yielding

$$\frac{1}{2}\langle\delta\theta_L(\vec{x})\delta\theta_L(\vec{x})\rangle \approx \frac{2\pi(2k+1)\pi}{4A}\sum_{\vec{q}}\frac{1}{q^2}. \quad (17)$$

Thus, for a square sample with $L_x = L_y = L$, we have $\frac{1}{2}\langle\delta\theta\delta\theta\rangle \approx \frac{2k+1}{4}\ln(L/l_0)$, where $l_0 = \sqrt{\frac{\nu}{2\pi\bar{\rho}}}$ is the magnetic length which we use as a cutoff length scale for the (continuum) CS gauge theory. Thus the fluctuations in $\delta\theta$ in the order parameter will only lead to a *power law* reduction factor of the Josephson current amplitude, which is in a certain sense not too severe.

The more severe quantum fluctuations come from the factor $\frac{1}{2}\langle\delta\theta_L^{\rm CS}(\vec{x})\delta\theta_L^{\rm CS}(\vec{x})\rangle$. This quantity is a bit more complicated to evaluate, due to the presence of the azimuthal angular variable $\alpha(\vec{x}-\vec{y})$. For simplicity but without loss of generality, let us choose the point $\vec{x}$ to be in the middle of the left (L) subsystem, and let us use this point as origin O' (see Fig. 2).

Since $\theta^{\rm CS}(\vec{x}) \equiv m\int d^2y\alpha(\vec{x}-\vec{y})\rho_L(\vec{y})$, the fluctuations in question can be written as

$$\begin{aligned}\langle[\delta\theta^{\rm CS}(0)]^2\rangle &= m^2\int d^2x d^2y\alpha(-\vec{x})\alpha(-\vec{y})\langle\delta\rho(\vec{x})\delta\rho(\vec{y})\rangle \\ &= \frac{m^2}{A^2}\sum_{\vec{q}}\int d^2x\alpha(-\vec{x})e^{-i\vec{q}\cdot\vec{x}} \\ &\quad\times\int d^2y\alpha(-\vec{y})e^{i\vec{q}\cdot\vec{y}}\langle\delta\rho_{\vec{q}}\delta\rho_{-\vec{q}}\rangle. \end{aligned} \quad (18)$$

It is easy to show from Eq. (15) that the ρ-ρ correlator is given by

$$\langle\delta\rho_{\vec{q}}\delta\rho_{-\vec{q}}\rangle \approx \frac{q^2A}{4\pi m},$$

for small q (i.e., when $q \ll 1/l_0$). So we can write the $\theta^{\rm CS}(\vec{0})$ fluctuations as

$$\langle[\theta^{\rm CS}(\vec{0})]^2\rangle = \sum_{\vec{q}}\frac{mq^2}{4\pi A}\int d^2x\alpha(-\vec{x})e^{-i\vec{q}\cdot\vec{x}} \times\int d^2y\alpha(-\vec{y})e^{i\vec{q}\cdot\vec{y}}. \quad (19)$$

Let us now apply the following Green's theorem

$$\int d^2x(\psi\nabla^2\phi - \phi\nabla^2\psi) = \oint dl\left(\psi\frac{\partial\phi}{\partial n} - \phi\frac{\partial\psi}{\partial n}\right) \quad (20)$$

to calculate the integral $\int d^2x\alpha(-\vec{x})e^{-i\vec{q}\cdot\vec{x}}$. We choose a contour depicted in Fig. 2, and identify the functions $\psi(\vec{x}) = \alpha(-\vec{x})$ and $\phi(\vec{x}) = -\frac{1}{q^2}e^{-i\vec{q}\cdot\vec{x}}$. In Eq. (20), $\hat{n}$ denotes the normal direction of the boundary. This gives the relation

$$\int d^2x\alpha(-\vec{x})e^{-i\vec{q}\cdot\vec{x}} = \oint dl\,\alpha(-\vec{x})\frac{iq_n}{q^2}e^{-i\vec{q}\cdot\vec{x}}. \quad (21)$$

[From the periodic boundary condition for the $\vec{q}$ values, the term $\oint dl\,\phi\frac{\partial\psi}{\partial n}$ vanishes.]

There are three contributions to Eq. (21) (taking $L_x = L_y = L$):

$$\begin{aligned}\oint dl\alpha(-\vec{x})\frac{iq_n}{q^2}e^{-i\vec{q}\cdot\vec{x}} &= 2\frac{iq_y}{q^2}\int_{-L/2}^{L/2}dx\arctan\left(\frac{2x}{L}\right)e^{-i(q_xx+q_yL/2)} \\ &\quad -2\frac{iq_x}{q^2}\int_{-L/2}^{L/2}dy\arctan\left(\frac{2y}{L}\right)e^{-i(q_xL/2+q_yy)} - 2\pi\int_0^{L/2}dxe^{iq_xx}\frac{iq_y}{q^2}.\end{aligned} \quad (22)$$

From the periodic boundary condition, we have $q_xL = 2\pi n_x$ and $q_yL = 2\pi n_y$, with $n_x, n_y = \pm 1, \pm 2, \ldots$.

By repeated partial integration we find

$$\int_{-1}^{1}\arctan(x)e^{in_x\pi x}dx = (-1)^{n_x}\frac{i}{2n_x} + O\left(\frac{1}{{n_x}^4}\right), \quad (23)$$

thus, the first term in Eq. (22) is approximately

$$-\frac{q_y\pi(-1)^{n_x}}{q_xq^2}e^{-iq_yL/2}.$$

By direct integration, we find

$$-2\pi\int_0^{L/2}dxe^{iq_xx}\frac{iq_y}{q^2} = \frac{2\pi q_y}{q^2q_x}(e^{iq_xL/2} - 1).$$

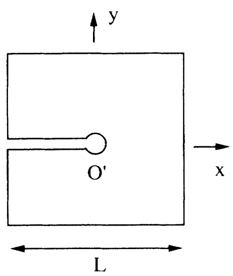

FIG. 2. The geometry for evaluating the contour integrals involved in the $\frac{1}{2}\langle\delta\theta_L^{\rm CS}(\vec{x})\delta\theta_L^{\rm CS}(\vec{x})\rangle$ calculations. The center position O' denotes the alternative origin for this calculation.

Thus, the entire contour integral has the form

$$\oint dl\alpha(-\vec{x})\frac{iq_n}{q^2}e^{-\vec{q}\cdot\vec{x}} \approx \frac{\pi(-1)^{n_x}(-1)^{n_y}}{q^2}\left\{\frac{q_x}{q_y}-\frac{q_y}{q_x} + \frac{2q_y(-1)^{n_y}}{q_x}[1-(-1)^{n_x}]\right\}, \quad (24)$$

where we have used the notation $q_xL/2 = n_x\pi$ and $q_yL/2 = n_y\pi$. We see from Eq. (19) that the Chern-Simons phase fluctuation integral can be rewritten as

$$\langle[\delta\theta^{\rm CS}(\vec{0})]^2\rangle \approx \sum_{\vec{q}}\frac{m\pi}{4q^2A}\left|\frac{q_x}{q_y}-\frac{q_y}{q_x} + \frac{2q_y(-1)^{n_y}}{q_x}[1-(-1)^{n_x}]\right|^2 . \quad (25)$$

From Eq. (25) we can derive a *lower* bound for the Chern-Simons phase fluctuations by replacing the above sum with one over $\frac{m\pi}{4q^2A}[(\frac{q_x}{q_y})^2+(\frac{q_y}{q_x})^2-10]$

$$\langle[\delta\theta^{\rm CS}(0)]^2\rangle \geq \sum_{\vec{q}}\frac{m\pi}{4q^2A}\left[\left(\frac{q_x}{q_y}\right)^2+\left(\frac{q_y}{q_x}\right)^2-10\right],$$

$$\langle[\delta\theta^{\rm CS}(0)]^2\rangle \geq \int\frac{d^2q}{2\pi^2}\frac{m\pi}{4q^2}\left[\left(\frac{q_x}{q_y}\right)^2+\left(\frac{q_y}{q_x}\right)^2-10\right]. \quad (26)$$

Since we can write the above integrals in the form

$$\int\frac{d^2\vec{q}}{q^2}\frac{q_i{}^2}{q_j} = \int d^2\vec{q}\left(\frac{1}{q_j{}^2}-\frac{1}{q^2}\right)$$

[where $i = x,y$, $j = x,y$, and $i \neq j$] we can integrate Eq. (26) as follows:

$$\int\frac{d^2\vec{q}}{q_i{}^2} \approx \frac{L}{l_0},$$

$$\int\frac{d^2\vec{q}}{q^2} \approx 2\pi\ln\left[\frac{L}{l_0}\right],$$

where l_0 is the cutoff length of the continuum theory and L is the length of one side of the square sample. Using these integrals we find the lower bound of the CS phase oscillations to be given by

$$\langle[\delta\theta^{\rm CS}(0)]^2\rangle \geq \frac{m}{16\pi}\left(2\frac{L}{l_0}+16\pi\ln\left[\frac{L}{l_0}\right]\right). \quad (27)$$

Thus, we see that the Josephson current amplitude I_0 is severely suppressed by quantum fluctuations of the CS phase angle $\theta^{\rm CS}$, with the final magnitude of order

$$I_0 \approx \frac{2eTE_0\bar{\rho}A}{\hbar}\exp\left(-\frac{2k+1}{4\pi}L/l_0\right). \quad (28)$$

This equation is the central result of this paper. Unfortunately, as we have seen, it is a negative result.

IV. DISCUSSIONS

Several remarks concerning our central result Eq. (28) are in order. Althougth this result is derived from a zero temperature calculation, we expect on physical grounds that it remains valid for any $k_BT < \Delta(\nu)$, where $\Delta(\nu)$ is the quasiparticle gap energy for $\nu = 1/(2k+1)$.

As we mentioned before, the Josephson current results from a bulk effect, as opposed to an edge tunneling effect, which governs the dissipative (or resistive) part of the coupling between the two subsystems.[9]

In a physically intuitive way, we may interpret our result as follows. The fundamental tunneling process between the L and R subsystems is that of electrons (fermions). In the case of superconductors, since the effective boson is a Cooper pair which consists of two electrons, one can readily write down a tunneling Hamiltonian for the Cooper pairs by a second order electron tunneling process, which gives a similar tunneling Hamiltonian whose tunneling amplitude is the square of that of the electron. In the case of FQHE state, the effective boson consists of an electron plus $2k+1$ flux quanta. These flux quanta cannot "tunnel" with the tunneling electron in a simple way. Leaving them behind creates phase fluctuations which severely suppress the Josephson effect.

We would like to remark that our conclusion of the severe quantum fluctuational suppression of the Josephson effect in the "side-by-side" geometry may not be easily generalizable to the vertically coupled double-layer geometry. In the latter case, the coupled 2DEG subsystems develop Coulomb correlations which may "lock" the Chern-Simons phase fluctuations enough to enable the Josephson effect to be measurable. Indeed, Wen and Zee[10] and Ezawa and Iwazaki[11] among others have made such a prediction. This view, however, is challenged by other authors recently.[12] It is not the task of the present paper to resolve this debate for the double-layer geometry. We present calculations which show unambiguously that in a single-layer geometry with a tunnel barrier, the Josephson effect is suppressed exponentially due to Chern-Simon phase fluctuations.

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