Here we briefly review the fundamentals of thermodynamics as discussed in the first two chapters of Callen. There are seven problems in all. Each is worth ten points.

1. **Callen 1.8-5**
The energy of a particular system, of one mole, is given by

\[ U = AP^2V \]  

where \( A \) is a positive constant with dimensions of inverse pressure. Find the equation of adiabats in the \( PV \) plane.

2. **Callen 1.8-7**
Two moles of a particular single-component system are found to have a dependence of internal energy \( U \) on pressure and volume given by

\[ U = APV^2 \]  

(for \( N = 2 \)).

Note that doubling the system doubles the volume, energy, and mole number, but leaves the pressure unaltered. Write the complete dependence of \( U \) on \( P,V \), and \( N \) for arbitrary mole number.

3. **Callen 2.2-4 and 2.2-5**
(a) Find the three equations of state for a system with the fundamental equation

\[ u = \left( \frac{\theta}{R} \right) s^2 - \left( \frac{R\theta}{v_0^2} \right) v^2 \]  

and show that, for this system, \( \mu = -u \).

(b) Express the chemical potential \( \mu \) as a function of temperature \( T \) and pressure \( P \) for this system.
4. Adiabats – one more time. From Callen, 2.2-9

We have seen for a single component ideal gas has adiabats of the form:

\[ pV^k = \text{const}, \]  

(4)

where \( k \) is some positive, dimensionless constant.

Using this result show that

\[ U = \frac{1}{k-1}pV + Nh \left( \frac{pV^k}{N^k} \right), \]

(5)

where \( h(x) \) is an unknown function. You would have to be given more information about this particular material to determine \( h \).

**Hint:** Convince yourself that \( pV^k \) is a function of only \( S \) (entropy) and \( N \) (number of particles). From the equation of state Eq. 4 you can then determine the fundamental relation Eq. 5 up to the unknown function as shown above.

5. Callen 2.6-4

Two systems with equations of state

\[
\frac{1}{T^{(1)}} = \frac{3}{2} R \frac{N^{(1)}}{U^{(1)}}
\]

(6)

\[
\frac{1}{T^{(2)}} = \frac{5}{2} R \frac{N^{(2)}}{U^{(2)}}
\]

(7)

are separated by a diathermal wall. There respective mole numbers are \( N^{(1)} = 2 \) and \( N^{(2)} = 3 \). The initial temperatures are \( T^{(1)} = 250K \) and \( T^{(2)} = 350K \). What are the values of the internal energies \( U^{(1,2)} \) after equilibrium has been established? What is the equilibrium temperature?

6. Callen 2.8-2

A two-component gaseous system has a fundamental equation of the form

\[
S = AU^{1/3}V^{1/3}N^{1/3} + \frac{BN_1N_2}{N},
\]

(8)

where \( N = N_1 + N_2 \) and where \( A, B \) are positive constants. A closed cylinder of total volume \( 2V_0 \) is separated into two equal sub volumes by a rigid diathermal partition permeable only to the first component [i.e. molecules of type (1)]. One mole of the first component, at a temperature \( T_l \) is introduced in the left-hand sub volume, and a mixture of \( 1/2 \) mole of each component, at temperature \( T_r \) is introduced into the right sub volume. Find the equilibrium temperature \( T \) and the mole numbers in each volume when the system has come to equilibrium, assuming that \( T_r = 2T_l = 400K \) and that \( 37B^2 = 100A^3V_0 \). Neglect the heat capacity of the walls of the container.
7. Preparation for Chapter three – A Generalized Euler Theorem:

In this problem you will examine a purely mathematical issue that will be important to us in the next chapter. Given that a function $f$ has the property:

$$f(\lambda^{p_1}x_1, \ldots, \lambda^{p_n}x_n) = \lambda^m f(x_1, \ldots, x_n) \quad (9)$$

show that

$$\sum_{j=1}^{n} p_j x_j \frac{\partial f}{\partial x_j} = mf(x_1, \ldots, x_n). \quad (10)$$