Physics 187
Homework #3
Due: Monday April 30th.

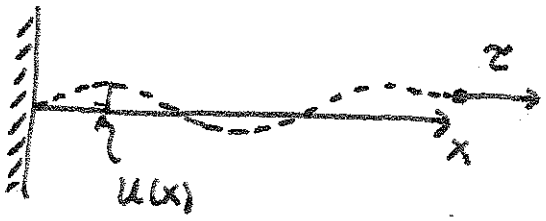
1. Jackson Problem 6. Chapter 3

2. Jackson Problem 9. Chapter 3

3. Jackson Problem 1. Chapter 4

4. Jackson Problem 4. Chapter 4.

5. We studied flexible polymers in Chapter 3. There is another limit in which the polymer is nearly straight ($L \ll l_p$) where it is also easy to calculate results. In this problem you will calculate the force vs. extension curve at temperature T for such a chain.



Under tension τ the Hamiltonian of the chain is

$$H = \int_0^L dx \left\{ \frac{1}{2} \kappa (\partial_x^2 u)^2 + \frac{1}{2} \tau (\partial_x u)^2 \right\}$$

bending modulus $\kappa = l_p k_B T$

a) Show that the length of the chain is

$$L_0 - L = \frac{1}{2} \int_0^L dx \, (\partial_x u)^2$$

where L_0 = length of the chain when completely stretched out.

b) Diagonalize the Hamiltonian in the Fourier basis:

$$u(x) = \sum_q u_q \sin(qx) \; ; \quad q = \frac{n\pi}{L}, \; n = 1, 2, \ldots$$

c) Use your result from part b to show that

$$\langle L \rangle = L_0 - \frac{k_B T L^2}{6K} + \frac{k_B T L^4}{90 K^2 \tau} + \ldots$$

d) Use your result from part c to identify the spring constant k_{eff} of the nearly straight chain.

In other words compute k_{eff} from

$$\tau = k_{eff} \, \delta L$$

(τ: applied force; δL: change in length.)

For more information, please see F.C. Mackintosh et al., PRL 75, 4425 (1995).

6. We studied the helix coil model for polymer conformations in class. This is an approach to the statistical physics of so-called secondary structure of proteins.

a) Using that model show that the correlation function

of order at sites s_j and s_{j+n} is given by:

$$\langle s_j s_{j+n} \rangle = Tr\left[T^{N-n} \sigma_z T^n \sigma_z \right]$$

for a polypeptide of N monomers. You may use periodic boundary conditions. Here $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ is the $\hat{z}$ Pauli spin matrix.

b) For the special case that $h=0$ (class notation) or $s=1$ (book notation) that

$$\langle s_j s_{j+n} \rangle = \tanh^n(J/k_BT) \quad \text{for } n \geq 0.$$

c) What is the correlation length in monomer units? Compare to our discussion of persistence length to get some insight.